Any $\mathcal{N}=2$ 4d SCFT

① Identify $sl(2) \times \widehat{sl(2)} \subset sl(4|2)$

commutes with Q — Q-exact

② Show that {local op representing Q-cohomology classes} lie in $\mathbb{R}^2 \subset \mathbb{R}^4$

≡ "Schur operators" ≡ $\boxed{\text{Operators contributing to Schur limit of SCI}}$

Rq:

2d QFT with global $SL(2,\mathbb{C})$ ($\to$ realification $sl(2)\times sl(2)$). The local meromorphic operators define a chiral algebra

[ex: in 2d CFT, Virasoro algebra, current algebra, ...]

* For the same construction to work in other dimensions, it is sufficient to find $sl(2)\times sl(2)$ inside the global sym.
* For it to be non-trivial, the two $sl(2)$ need not to be related by conjugation (otherwise there is no good "chirality" notion)

Rq: Schur operators satisfy a chiral OPE

$$\mathcal{O}_1(z,\bar z)\,\mathcal{O}_2(0,0) = \sum_k \frac{\lambda_{12k}}{\underbrace{z^{h_1+h_2-h_k}}_{\text{no } \bar z \text{ dependence!}}}\,\mathcal{O}_k(0,0) + \{Q,\dots]$$

SCFT / Chiral algebra correspondence $\boxed{\chi : \text{4d SCFT} \longrightarrow \text{2d Chiral Alg}}$

Central charges: $\boxed{c_{2d} = -12\, c_{4d}}$ [Hence $\mathcal{T}$ unitary $\Rightarrow \chi[\mathcal{T}]$ non unitary]

In a lot of examples, $\chi[\mathcal{T}]$ is a $\mathcal{W}$-algebra (ie chiral algebra with finite set of generators)

Two universal features of χ:

① Affine enhancement of global flavor symmetry $k_{2d} = -\frac{1}{2}k_{4d}$

② Virasoro enhancement of global conformal symmetry $c_{2d} = -12\, c_{4d}$.

Any 6d $\mathcal{N}=(2,0)$ SCFT

① Identify $\mathfrak{sl}(2) \times \widehat{\mathfrak{sl}(2)} \subset \mathfrak{osp}(8^*|4)$ and twisted $\mathbb{Q}_{i=1,2,3,4}$ that are nilpotent

$\mathfrak{sl}(2)$: commutes with $\mathbb{Q}_i$ — $\widehat{\mathfrak{sl}(2)}$: $\mathbb{Q}_i$-exact

② Consider simultaneous cohomology of all the $\mathbb{Q}_i$. They are meromorphic operators $[\mathcal{O}(z,\bar{z})]_{\mathbb{Q}} \rightsquigarrow \mathcal{O}(z)$ that enjoy meromorphic OPE.

Rq Representation theory of (2,0) SCA known. (5 quantum numbers + scaling dimension).

↳ 4 series $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$ of "short" representations.

Ex $\boxed{\text{Abelian (2,0) theory (ie free tensor multiplet)}} \rightarrow \boxed{\text{u(1) affine current algebra}}$.

Chiral algebras of interacting (2,0) theories

Conjecture: $\boxed{\text{chiral algebra of type } \underline{\mathfrak{g}} \text{ (2,0)} = \mathcal{W}_{\underline{\mathfrak{g}}}\text{-algebra}}$

central charge $c_{2d} = 4 \dim(\underline{\mathfrak{g}})\, h^\vee(\underline{\mathfrak{g}}) + \operatorname{rank}(\underline{\mathfrak{g}})$

Superconformal index $\mathcal{I}(p,q,s,t) = \operatorname{Tr}(-1)^F e^{\beta\{\dots\}} q^{E-R} p^{h_2-h_3+2r} t^{R-r} s^{h_2+h_3}$

$\downarrow$ unrefining $t=1$, $p=1$

$\boxed{\text{Witten index of 2d chiral algebra} = \text{Unrefined index } \mathcal{I}(q,s)}$

|| (left) For $\mathcal{W}_{\underline{\mathfrak{g}}}$-algebra, this is

$$\mathrm{PE}\left[\frac{\sum_i q^{s_i}}{1-q}\right]$$

where the s_i are the degrees of the invariants

|| (right) Can be computed directly in some cases (see eg. eq (3.22) in 1307.7660)

$\boxed{\mathcal{W}\text{- algebras}}$ (9210010)

A ==meromorphic CFT== consists of:

- a "characteristic Hilbert space" $\mathcal{H}$
- a map $|\Psi\rangle \longrightarrow V(|\Psi\rangle, z)$ called "vertex-operator map"
- a distinguished state $|L\rangle$ with $T(z) = V(|L\rangle, z) =$ stress-energy tens. (whose modes satisfy the Virasoro algebra).

with technical assumptions on the map V.

A quantum $\mathcal{W}$-algebra is a MCFT such that:

- $\mathcal{H}$ contains a finite # of states $|i\rangle$ (including $|L\rangle$) whose $W^{(s_i)}(z) = V(|i\rangle, z)$ are quasi-primaries with conformal dimension $s_i \in \mathbb{N}$
- $T(z) = W^{(2)}(z)$
- The entire space of fields is spanned by normal ordered products of the $W^{(s_i)}$ and their derivatives.

Any $\mathcal{W}$-algebra has a central charge c coming from TT OPE.

- If the theory exists only for isolated values of c $\rightarrow$ EXOTIC
- Otherwise, $\rightarrow$ GENERIC